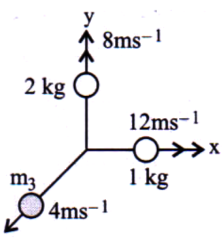
WEEKLY TEST TYM TEST - 19 Balliwala
SOLUTION Date 01-09-2019

[PHYSICS]

1.

The situation of the problem is as shown in the figure. According to law of conservation of linear momentum



$$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$$

$$\therefore \quad \vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$$

Here,

$$\vec{p}_1 = (1\text{kg})\,(12\text{ ms}^{-1})\hat{i} = 12\,\hat{i}\text{ kg ms}^{-1}$$

$$\vec{p}_2 = (2\text{ kg})\,(8\text{ ms}^{-1})\,\hat{j} = 16\hat{j}\text{ kg ms}^{-1}$$

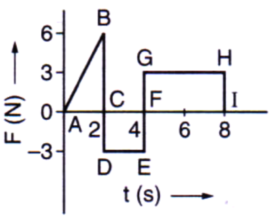
$$\therefore \quad \vec{p}_3 = -(12\,\hat{i} + 16\hat{j})\text{ kg ms}^{-1}$$

The magnitude of $\vec{p}_3$ is :

$$p_3 = \sqrt{(12)^2 + (16)^2} = 20\text{ kg ms}^{-1}$$

$$\therefore \quad m_3 = \frac{p_3}{v_3} = \frac{20\text{ kg ms}^{-1}}{4\text{ ms}^{-1}} = 5\text{kg}$$

2.

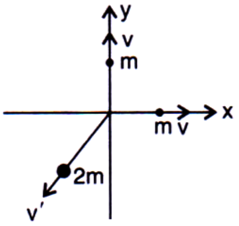


Change in momentum = Area under *F-t* graph in that interval

= Area of ΔABC – Area of rectangle *CDEF* + Area of rectangle *FGHI*

$$= \frac{1}{2}\times 2\times 6 - 3\times 2 + 4\times 3 = 12\text{ N s}$$

3.



Let $\vec{v'}$ be velocity of third piece of mass $2m$. Initial momentum, $\vec{P_i} = 0$ (As the body is at rest). Final momentum,

$\vec{P_f} = mv\hat{i} + mv\hat{j} + 2m\vec{v'}$

According to law of conservation of momentum

$\vec{p_i} = \vec{p_f}$

$0 = mv\hat{i} + mv\hat{j} + 2m\vec{v'}$

$\vec{v'} = -\frac{v}{2}\hat{i} - \frac{v}{2}\hat{j}$

The magnitude of $\vec{v'}$ is

$v' = \sqrt{\left(-\frac{v}{2}\right)^2 + \left(-\frac{v}{2}\right)^2} = \frac{v}{\sqrt{2}}$

Total kinetic energy generated due to explosion

$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)v'^2$

$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)\left(\frac{v}{\sqrt{2}}\right)^2 = mv^2 + \frac{mv^2}{2}$

$= \frac{3}{2}mv^2$

4.

Given that,

$\vec{F} = (2t\hat{i} + 3t^2\hat{j})$ and $\vec{a} = 2t\hat{i} + 3t^2\hat{j}$

Hence, $v = \int_0^t a\,dt = t^2\hat{i} + t^3\hat{j}$

$\therefore \quad P = \vec{F}\cdot\vec{v} = 2t\cdot t^2 + 3t^2\cdot t^3 = 2t^3 + 3t^5$

5.

Here, $m_1 = m, m_2 = 2m$

$u_1 = 2\text{ m/s}, \quad u_2 = 0$

Coefficient of restitution, $e = 0.5$

Let v_1 and v_2 be their respective velocities after collision.

Applying the law of conservation of linear momentum, we get,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$\therefore \quad m \times 2 + 2m \times 0 = m \times v_1 + 2m \times v_2$

or $\quad 2m = mv_1 + 2mv_2$

or $\quad 2 = (v_1 + 2v_2) \quad \ldots(i)$

By definition of coefficient of restitution,

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

or $\quad e(u_1 - u_2) = (v_2 - v_1)$

$0.5(2 - 0) = (v_2 - v_1)$

$1 = v_2 - v_1 \quad \ldots(ii)$

Solving equations (i) and (ii), we get,

$$v_1 = 0\text{ m/s}, v_2 = 1\text{ m/s}$$

6.

According to conservation of momentum

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v,$$

where v is common velocity of the two bodies.

$m_1 = 0.1\,\text{kg}$, $m_2 = 0.4\,\text{kg}$

$v_1 = 1\,\text{m/s}$, $v_2 = -0.1\,\text{m/s}$

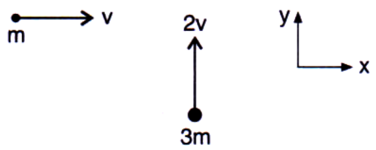
$\therefore\ 0.1 \times 1 + 0.4 \times (-0.1) = (0.1 + 0.4)v$

or $0.1 - 0.04 = 0.5v,$

$$v = \frac{0.06}{0.5} = 0.12\,\text{m/s}.$$

Hence, distance covered = $0.12 \times 10 = 1.2$ m

7.



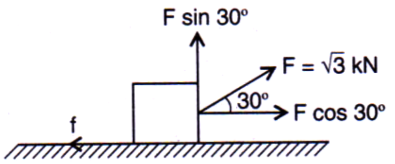
According to conservation of momentum, we get

$$mv\hat{i} + (3m)2v\hat{j} = (m + 3m)v'$$

where v' is the final velocity after collision

$$v' = \frac{1}{4}v\hat{i} + \frac{6}{4}v\hat{j} = \frac{1}{4}v\hat{i} + \frac{3}{2}v\hat{j}.$$

8.



The component of applied force F in the direction of motion is $F\cos 30^\circ$.

The work done by the applied force is,

$$W = (F\cos 30^\circ)S = \sqrt{3} \times 10^3 \times \frac{\sqrt{3}}{2} \times 10\,\text{J}$$

$$= 15 \times 10^3\,\text{J} = 15\,\text{kJ}.$$

9.

Mass of water falling/second = 15 kg, $h = 60$ m

$g = 10\,\text{m/s}^2$, loss = 10%, *i.e.*, 90% is used

Power generated $= 15 \times 10 \times 60 \times 0.9 = 8100$ W $= 8.1$ kW

10. $mv = Mv'$ or $v' = \left(\frac{m}{M}\right) v$

Total KE of the bullet and the gun $= \frac{1}{2} mv^2 + \frac{1}{2} Mv'^2$

$$\text{Total KE} = \frac{1}{2} mv^2 + \frac{1}{2} M \cdot \frac{m^2}{M^2} v^2$$

$$\text{Total KE} = \frac{1}{2} mv^2 \left[1 + \frac{m}{M}\right]$$

or $1.05 \times 1000 \text{ J} = \left[\frac{1}{2} \times 0.2\right]\left[1 + \frac{0.2}{4}\right] v^2$

or $v^2 = \frac{4 \times 1.05 \times 1000}{0.1 \times 4.2} = (100)^2$;

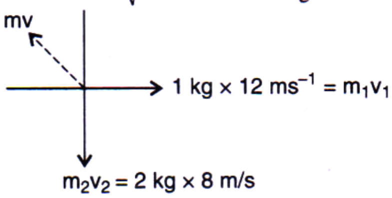
$\therefore$ $v = 100 \text{ ms}^{-1}$

11.

When an explosion breaks a rock, by the law of conservation of momentum, initial momentum which is zero, is equal to total momentum of three pieces.

Total momentum of the two pieces 1 kg and 2 kg

$$= \sqrt{12^2 + 16^2} = 20 \text{ kg m s}^{-1}$$



The third piece has the same momentum and in the direction opposite to the resultant of these two momenta.

$\therefore$ Momentum of the third piece $= 20 \text{ kg ms}^{-1}$;

Velocity $= 4 \text{ ms}^{-1}$

$\therefore$ Mass of the 3rd piece $= \frac{mv}{v} = \frac{20}{4} = 5$ kg.

[CHEMISTRY]

22.

$P_1 = 100, V_1 = 100, V_2 = 110$

$P_2V_2 = P_1V_1 \Rightarrow P_2 = \frac{100 \times 100}{110} = \mathbf{90.9}$

23.

$$\frac{r_{CH_4}}{r_{SO_2}} = \frac{\left(\frac{V}{t}\right)_{CH_4}}{\left(\frac{V}{t}\right)_{SO_2}} = \sqrt{\frac{M_{SO_2}}{M_{CH_4}}}$$

$$\frac{\frac{200}{20}}{\frac{400}{t}} = \sqrt{\frac{64}{16}} = 2$$

$\frac{t}{40} = 2 \quad \Rightarrow \quad \mathbf{t = 80 \text{ sec}}$